Solutions

3.1: Systems of Two Equations in Two Unknowns

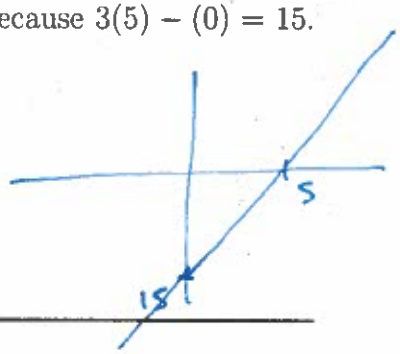
Definition 1. A linear equation in two unknowns is an equation that can be written in the form

$$ax + by = c$$

with a, b and c being real numbers. The number a is called the coefficient of x and b is called the coefficient of y. A solution of an equation consists of a pair of numbers for x and y that satisfy the equation.

Example 1. In the linear equation $3x - y = 15$, the coefficients are $a = 3$ and $b = -1$. The point $(x, y) = (5, 0)$ is a solution, because $3(5) - (0) = 15$. The graph represents all solutions to the equation.

$3x - y = 15 \Rightarrow y = 3x - 15$



Example 2. (Solving a system of two equations graphically and algebraically) Find all solutions (x, y) of the following system of two equations:

$$x + y = 3 \quad x - y = 1.$$

Algebraically

$x + y = 3$

$+ \quad x - y = 1$

$2x = 4$

$x = 2$

then

$2 + y = 3 \Rightarrow y = 1$

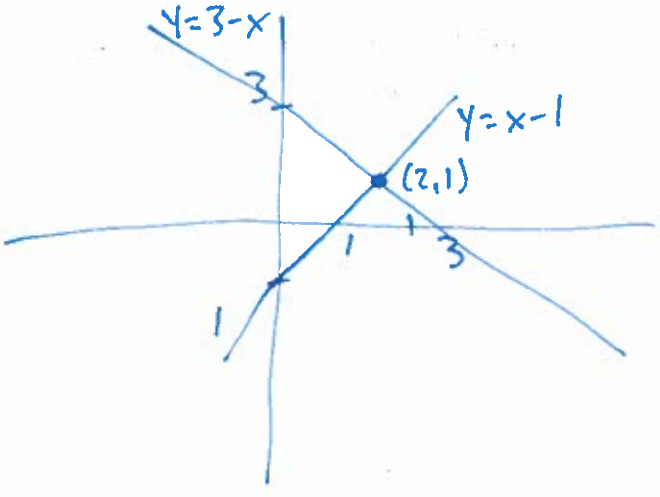
Solution $= (x, y) = (2, 1)$

Graphically

You can use calculator !!

$x + y = 3 \Rightarrow y = 3 - x$

$x - y = 1 \Rightarrow y = x - 1$



Example 3. Solve the system both graphically and algebraically and verify that you get the same example.

$$3x + 5y = 0 \quad 2x + 7y = 1.$$

$2(3x+5y=0)$

$-3(2x+7y=1)$

$-11y = -3$

$y = \frac{3}{11}$ $\quad (x,y) = \left(\frac{-5}{11}, \frac{3}{11}\right)$

$\Downarrow$

$x = \frac{-5}{11}$

Graphically: Use calculator.

Example 4. Solve the system

$$x - 3y = 5 \quad -2x + 6y = 8.$$

$2(x-3y=5)$

$+(-2x+6y=8)$

$0 = 18$

No solution. Parallel Lines

Inconsistent System.

Example 5. Solve the system

$$x + y = 2 \quad 2x + 2y = 4.$$

$2(x+y=2)$

$-(2x+2y=4)$

$0+0=0$

Infinitely Many Solutions.

Exact Same Line.

Redundant System.

Example 6. (Blending) Acme Baby Foods mixes two strengths of apple juice. One quart of Beginner's juice is made from 30 fluid ounces of water and 2 fluid ounces of apple juice concentrate. One quart of Advanced juice is made from 20 fluid ounces of water and 12 fluid ounces of apple juice concentrate. Every day Acme has available 30,000 fluid ounces of water and 3,600 fluid ounces of concentrate. If the company wants to use all the water and concentrate, how many quarts of each type of juice should be mixed?

B = Beginner's Juice

A = Advanced Juice

~~# ounces of water~~

~~# ounces of juice concentrate~~

	B (x)	A (y)	Total
Water	30	20	30,000
Juice Concentrate	2	12	3,600

x = # of quarts of B

y = # of quarts of A

So $30x + 20y = 30{,}000$

and $2x + 12y = 3{,}600$

So, dividing top by 10, $3x + 2y = 3{,}000$

$$-\tfrac{3}{2}(2x + 12y = 3{,}600)$$

$$-16y = -2{,}400$$

$y = 150$. So $x = 900$.

Acme Baby Foods should make 900 quarts of Beginner's juice and 150 quarts of Advanced juice.

Example 7. A medieval alchemist's love potion calls for a number of eyes of newt and toes of frog, the total being 20, but with twice as many ~~next~~ newt eyes as frog toes. How many of each is required?

X = # of eyes of newt

Y = # of toes of frog

$$X + Y = 20$$
$$-(X - 2Y = 0)$$

$3Y = 20 \Rightarrow Y = \frac{20}{3} = 6\frac{2}{3}$

and $X = 13\frac{1}{3}$.

---

8. (Equilibrium Price) The demand for refrigerators in West Podunk is given by

$$q = -\frac{p}{10} + 100$$

where q is the number of refrigerators that the citizens will buy each year if they are priced at p dollars each. The supply is given by

$$q = \frac{p}{20} + 25$$

where q is the number of refrigerators the manufacturers will be willing to ship into town each year is they are priced at p dollars each. Find the equilibrium price and the number of refrigerators that will be sold at that price.

$$\frac{p}{10} + q = 100$$
$$-\left(\frac{-p}{20} + q = 25\right)$$

$\frac{3p}{20} = 75 \Rightarrow p = 500$ and $q = 50$.

So the equilibrium price is \$500 and

50 refrigerators will sell at this price.